A FIELD TEST OF THE FOUR-THIRDS LAW OF HORIZONTAL DIFFUSION IN THE OCEAN

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by

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ABSTRACT

This paper reports the results of field tests of Richardson's "four-thirds law" of horizontal diffusion, which relates the horizontal diffusion coefficients to particle separation, or eddy scale.

A U. S. Navy photo-reconnaissance aircraft was used to gather data of diffusing patterns of rectangular paper sheets on the ocean surface and of current crosses nine feet below the surface. The data were gathered in Monterey Bay, California in water thirty-six fathoms deep in an area approximately one and one-half miles from the shore. The scale covered was from seventy to four hundred and sixty feet. A two-particle analysis suggested by Richardson and Stommel was applied. The four-thirds law was found to be applicable to the horizontal diffusion analyzed in this investigation.

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1. Richardson's diffusion equation and the meaning of the "four-thirds law."

Richardson (1926) analyzed the data from seven very rough observations of atmospheric diffusion covering a scale from one meter to ten kilometers and deduced "that the rate of diffusion increases with the distance apart." That is, as the distance between particles increases, so does their time rate of separation or diffusion.

If this conclusion is correct the classical Fickian diffusion equation

$$\frac{\partial v}{\partial t} = K \frac{\partial^2 v}{\partial x^2}$$

where v=concentration of diffusing substance, K=diffusivity (a measure of the rate of diffusion), x=position, and t=time, cannot be applied to atmospheric diffusion, for it predicts a constant rate of diffusion at each point at any time. More specifically, as was pointed out by Stommel (1949), if one tries to determine the probability of two particles initially a distance l_0 apart being a distance l_1 apart at a later time, the Fickian equation leads to the result

$$P(l_0, l_1) = \frac{1}{2\sqrt{\pi K t}} \exp\left(-\frac{(l_1 - l_0)^2}{4Kt}\right) .$$

This result states that the probability of a pair of particles a distance l_0 apart at time t=o being a distance l_1 apart at a later time t depends only upon $(l_1 - l_0)^2$ and not upon either l_0 or l_1. This result is in direct contradiction to the atmospheric observations, and there seems to be no way in which the Fickian equation can be modified to overcome the discrepancy.

To resolve this problem Richardson (1926) postulated the following diffusion equation

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial \ell}\left[F(\ell)\frac{\partial q}{\partial \ell}\right]$$

where ℓ is the separation between two particles in a cluster and is called the "neighbor separation." The number of particles which have neighbors with neighbor separations between ℓ and $\ell + \partial \ell$ is, by definition, $q(\ell)\partial\ell$, where $q(\ell)$ is the "neighbor concentration." $F(\ell)$ is analogous to the diffusivity K in the Fickian equation and is called the "neighbor diffusivity." The result is a diffusion equation in which the neighbor diffusivity $F(\ell)$ is some function of neighbor separation ℓ.

The fundamental reason, then, that Richardson introduced neighbor separations to describe diffusion in the atmosphere was that he had observed that the rate of diffusion increased with increasing particle separation and that the classical Fickian equation does not describe this phenomena.

From the same seven measurements of atmospheric diffusion Richardson induced that the neighbor diffusivity was related to the neighbor separation in the following manner:

$$F(\ell) = k\ell^{4/3}$$

where k is a constant. This equation expresses what has become known as the "four-thirds law." This empirical relationship has received theoretical support from the work of Kolmogoroff (1941), Obukov (1941), Onsager, Weisaecker and Heisenburg (1948) and others.

2. Richardson's method for determining neighbor diffusivity as a function of neighbor separation.

If neighbors at initial neighbor separation ℓ_0 are set free to diffuse, after a time interval T their neighbor separation will be ℓ_1. If T is such that $(\ell_1 - \ell_0)$ is only a small fraction of ℓ_0 Richardson's diffusion equation may be written as

$$\frac{\partial q}{\partial t} = F(\ell_0) \frac{\partial^2 q}{\partial \ell^2}$$

since the scale of phenomena, ℓ, will be practically the same throughout the time interval. A solution of this equation is

$$q(\ell_1) = \frac{\text{CONST}}{\sqrt{T}} \exp\left(-\frac{(\ell_1 - \ell_0)^2}{4TF(\ell_0)}\right)$$

which expresses the distribution of $q(\ell_1)$ about the mean separation after time T. This distribution is Gaussian, so that the standard deviation of ℓ_1 from the mean ℓ_0 is

$$\sqrt{2TF(\ell_0)} \quad .$$

Solving for $F(\ell_0)$ yields

$$F(\ell_0) = \frac{\overline{(\ell_1 - \ell_0)^2}}{2T}$$

with the bar indicating the mean of all pairs. Since, as pointed out by Richardson and Stommel (1948), it is difficult in practice to have ℓ_0 for each pair identical, a mean of the type $\frac{1}{2}(\ell_0 + \ell_1)$ can be computed as a measure of the scale, resulting in the following equation

$$F\left(\frac{1}{2}\overline{(\ell_1 + \ell_0)}\right) = \frac{\overline{(\ell_1 - \ell_0)^2}}{2T} \quad .$$

Because this equation was developed by Richardson (1926) based on a one-dimensional cluster formed by diffusion on a straight line from a point, Stommel and Richardson used the projections of a diffusing cluster of floats onto a straight line as measures of ℓ_0 and ℓ_1. A modification of this procedure, based on the suggestions of Ozmidov (1957), was used in the reduction of the data discussed in the present paper.

3. A brief history of experiments testing the applicability of the four-thirds law to horizontal diffusion in the ocean

Richardson and Stommel (1948) first tested the four-thirds law in the ocean, using parsnip floats as the diffusing particles in water about two meters deep. The power law which best fit their two resulting data points (at neighbor separations of 26.7 cm and 187.7 cm) was

$$F(\ell) = 0.0072\,\ell^{1.4} \quad (\text{ft}^2/\text{sec}).$$

Stommel (1949) combined this earlier data with new data obtained using dye spots and paper markers and showed that the four-thirds law seemed to describe the observed diffusion up to a scale of 330 feet. Olson and Ichiye (1959) showed, by combining Olson's drift-card data with jointly obtained drift-bottle data and the data of Richardson and Stommel, that the relation

$$F(\ell) = 0.0025\,\ell^{4/3} \quad (\text{ft}^2/\text{sec})$$

fitted the whole series of data from .33 to 3.28×10^6 feet with a correlation coefficient of 0.993.

Ozmidov (1957) in an extensive series of tests in a basin of variable depth (14, 35, 45 and 57 cm) and in the sea at depths of 2 and 10 meters, found that the four-thirds law held only in the case of L greater

than 10h, where h is the water depth and L represents the maximum eddy size present contributing to the diffusion. This maximum eddy size was taken to be the distance from the diffusing cluster to the shoreline, in the case of the ocean experiments, and to be the maximum horizontal dimension of the basin in the basin experiments. These tests covered neighbor separations up to 20 meters. Pochapsky (1965) tested Ozmidov's theory of the dependence of the four-thirds law on depth and found no such depth dependence, but rather that the neighbor diffusity followed the four-thirds law directly. Ozmidov (1959) obtained horizontal diffusion data in the Pacific Ocean at scales from 1,000 to 5,000 feet using radar buoys as diffusing particles. Combining this data with his earlier (1957) small-scale data taken in the Caspian Sea he found that the relationship

$$F(\ell) = 0.001\,\ell^{4/3} \qquad (\mathrm{ft}^2/\mathrm{sec})$$

fit the data well.

Alsaffar (1966), with a limited number of measurements over a two to five foot scale at the mouth of the San Joaquin River, found that the least-squares fit to his data was

$$F(\ell) = 0.00424\,\ell^{1.345} \qquad (\mathrm{ft}^2/\mathrm{sec}).$$

Orlob (1959) combined his data with a summary of previous data compiled by E. A. Pearson (1957) and found that the relationship

$$F(\ell) = 0.001\,\ell^{4/3} \qquad (\mathrm{ft}^2/\mathrm{sec})$$

"fairly well establishes the over-all trend of data from scales of about 0.1 ft. to scales of more than 1,000 ft."

Other experiments, notably those using the diffusion of dye patches, have cast doubts on the applicability of the four-thirds law to oceanic turbulent diffusion. Isayeva and Isayev (1963) reported that "investigations from a ship in the open sea for scales up to 10^2 m show that the '4/3 law' is not valid...." Also, Foxworthy, Barsom and Tibby (1966) reported a series of dye diffusion experiments conducted in the near-shore coastal waters of southern California from 1963 to 1966 and concluded that "In more than 40 individual experiments conducted in the course of this investigation, no evidence has been found to substantiate the applicability of the four-thirds law which relates the horizontal diffusion coefficients to the approximate eddy scale."

The scale between 10 and 500 feet is investigated in this report. Pearson (1956), Parker (1961), Stommel (1949) and Gunnerson (1959) have all investigated horizontal diffusion in this range. The basic papers of Pearson and Parker were not available, although from summaries such as that of Wiegel (1964), their results give much higher values of diffusivity for a given scale than the results of Stommel or Gunnerson. Stommel's (1949) results have been discussed. Gunnerson's results were obtained by measuring the width of dye streams released in Santa Monica Bay, California in water 180 feet deep. As far as is known to the author, his investigation is the most extensive in this particular scale range; it resulted in the relationship

$$F(\ell) = 0.000051\,\ell^{4/3} \quad (\text{ft}^2/\text{sec}).$$

4. Objectives of the present test

The present test was designed to test the applicability of the four-thirds law to turbulent horizontal diffusion in the ocean at as large a

scale as practical, using photographic data-gathering techniques. The method of two-particle analysis was chosen instead of a dye method so as to remain within the confines of the technique originally suggested by Richardson to test his neighbor diffusion equation.

Another objective was to measure the diffusion at different depths, by means of current crosses, and to thus obtain a three-dimensional view of the horizontal diffusion process in the ocean. As far as the author has been able to determine, no previous attempts with this second objective in mind have been reported in the literature.

Results were desired on a scale not intensively investigated in previous particle-diffusion experiments. Therefore, a scale greater than 100 feet was aimed for. Most of the previous values of neighbor diffusivity arrived at from particle diffusion experiments were obtained from averages of 20 or less measurements. It was also desired to obtain as much data at each scale as practical so as to make the averaged results as statistically valid as possible.

5. Developing a data-gathering technique

The first attempts to develop a technique for gathering data using photographic methods were made in January 1967 using a Navy helicopter and hand-operated 35 mm and 4 x 5-inch speed-graphic cameras. The diffusing particles, 8 by 10.5-inch offset duplicating bond paper, substance 20, were dropped from the helicopter and photographs were taken of the resulting cluster of paper sheets on the water surface.

Many difficulties were found to be inherent in this method. The cluster, once on the water surface, was difficult to relocate if the helicopter traveled away from it. The pilot also had difficulty in positioning the helicopter so that the photographer was directly over the

cluster and could take a vertical picture, mainly because the pilot could not see the cluster when he was directly overhead, and so had no reference with which to maintain position.

From the photographs obtained after several flights the following conclusions were reached. At altitudes greater than 500 feet the image size of the 8 by 10.5-inch paper on 35 mm film was approximately the same size as the impurities in the developing solutions available, and consequently the paper could not be isolated from the impurities on the negative. The paper sheets could be seen in the 4 x 5-inch negatives taken with the speed-graphic up to an altitude of about 1,000 feet, although with some difficulty. This would enable clusters with diameters up to about 500 feet to be analyzed using paper of this size.

The main objection to the helicopter method was that the tilt of the camera at the time the photograph was taken could not be determined so that an accurate length scale could not be applied to the entire negative for analysis. If the helicopter were at an altitude of 1,000 feet and the camera were tilted 10^{o} to the vertical, a 7.5-percent scale error would occur on a particular azimuth even if an object of known size ($\leq$ 10' x 10') were introduced into the pattern. The pictures were taken, using both cameras alternately, by the author while leaning out of the open hatch of the helicopter while attached by a "gunner's sling" around his waist. This seemed to be the only practical method of obtaining vertical pictures of the water surface immediately below the aircraft, and a tilt of at least 10^{o} was considered to be unavoidable when using this photographic method.

Although a method using the 4 x 5-inch speed-graphic camera in combination with a helicopter is deemed feasible for gathering data on

diffusing particles, the errors inherent in the technique were unacceptable, particularly since a better method became available.

6. Method used in gathering diffusion data

A Navy RC-45J aircraft was used to obtain the data analyzed in this report. It is a standard SNB Beechcraft modified for photo reconnaissance with the CA-3-2-B reconnaissance camera and photo-navigational equipment. Plus X Kodak Aerecon Safety film 1B-A was used. A Blue-Minus filter served to increase the contrast between the water and the white paper and white plywood sheets used as diffusing particles. The filter had the effect of making the water appear clear and the white sheets black on the negatives. Twelve and six-inch focal lengths were available. The twelve-inch focal length was chosen so as to minimize the image distortion. The optical navigation system allowed the pilot to fly the aircraft almost directly over the diffusing pattern. The camera mounting enabled the tilt from the vertical to be kept within two degrees on all photographic runs.

Two patterns were to be laid from a forty-foot motor launch. After laying each pattern the launch would station itself one hundred yards from the pattern keeping it in the same relative position throughout the series of photographic runs. This would give the navigator a reference he could keep in sight at all times, as the pattern itself could not be seen unless the plane was almost over it. The first pattern was to consist of 25 (card index, 110 pound) paper sheets 30.5 by 25.5-inches, which were to be laid in a circular pattern, about 70 feet apart. This would result in a pattern about 540 feet in diameter. The advantages of the circular pattern were two-fold. It enabled one to obtain data on a range of scales from the distance between adjacent particles, about 70

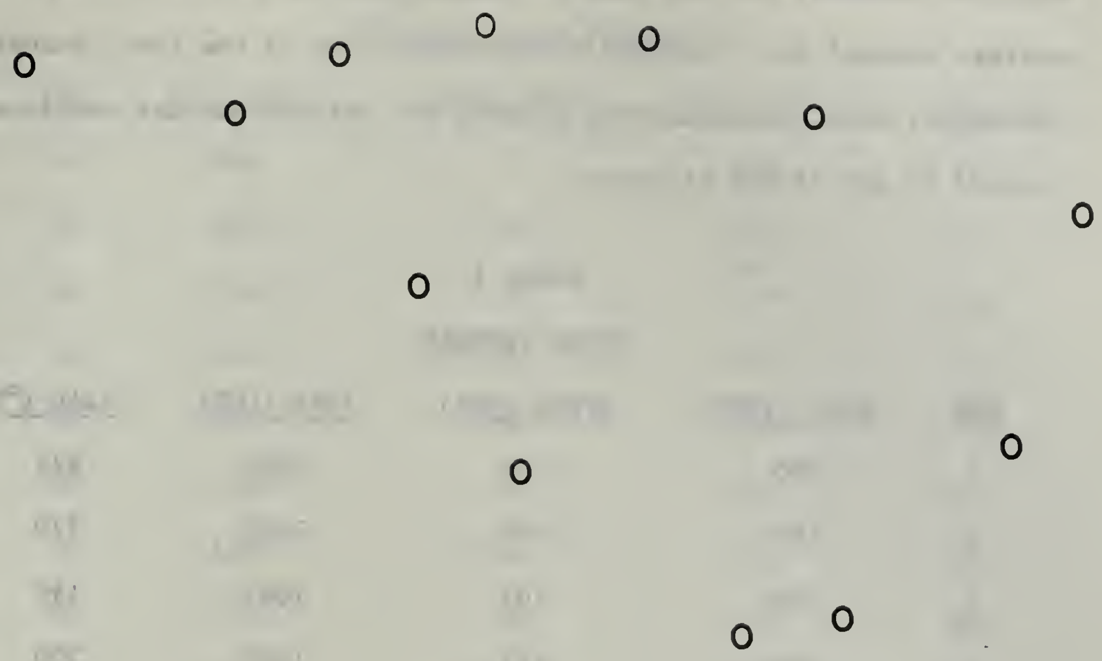
feet, to the diameter of the diffusing circle, about 540 feet. The circular pattern also made it easier to identify individual particles throughout a series of photographs.

The second pattern was to be identical to the first in geometry. However, in place of the paper sheets, current crosses at a depth of nine feet were to be used. A description of these current crosses with their attached floats and plywood sheets for identification can be found in Appendix I.

The measurements were taken on March 14, 1967 and began at 1036 A.M. Two patterns were laid in Monterey Bay at 36° 38.5'N 121° 53.5'W in 36 fathoms of water about one and one-half miles from the shore. The wind was from 210° true north at 8-10 knots. The water was almost isothermal to the bottom and the waves were less than a foot in height with little or no swell present. No strong currents are present in this area and the tidal currents are too weak and variable to be predicted.

The patterns that were actually laid from the launch did not contain as many particles as planned. The pattern of paper sheets consisted of only twelve particles and the current-cross pattern of thirteen particles. The patterns were initially of the shapes shown in Figure 1. Initially the long diameter of the pattern of paper sheets was about 371 feet and that of the pattern of current crosses about 445 feet. The diameters expanded to 555 feet and 512 feet respectively during the two series of photographic runs, each of which lasted about 35 minutes.

The pattern of paper sheets was laid first and eight runs were made across the pattern at time intervals varying from three to nine minutes as shown in Table I. On each run a series of pictures was taken at two-second intervals while passing over the pattern. Altitude, speed,



INITIAL PATTERN OF PAPER SHEETS
(1 inch = 66.6 feet)

INITIAL PATTERN OF CURRENT CROSSES
(1 inch = 80.7 feet)

Figure 1

magnetic heading, and the time of commencement of picture taking (to the nearest minute) were recorded on each run. Two of the runs, numbers five and eight, were unsatisfactory in that the pattern was not completely covered in any of the pictures.

TABLE I

PAPER PATTERN

RUN	ALT. (FEET)	SPEED (KTS)	TIME (LMT)	HEAD (^{o}M)
1	800	105	1036	225
2	1020	100	1039	270
3	1000	103	1043	190
4	1040	102	1048	200
5	1230	108	1052	192
6	1020	104	1101	195
7	1030	103	1105	198
8	1080	102	1109	198

The pattern of current crosses was laid after completing the runs over the first pattern. Ten runs were made, in the same manner as before, at time intervals of from three to seven minutes as shown in Table II. Run mumber seven was unsatisfactory due to the fact that the entire pattern was not included in any of the pictures taken on that run. The last two runs were made mainly to determine the maximum height at which the plywood markers could be seen in the photographs. They were clearly visible at 2,200 feet, the highest altitude reached.

TABLE II

CURRENT CROSS PATTERN

RUN	ALT. (FEET)	SPEED (KTS)	TIME (LMT)	HEAD (^{o}M)
1A	980	97	1119	197
2A	1020	100	1122	197
3A	1030	102	1129	198
4A	1060	108	1134	196
5A	1200	107	1137	198
6A	1200	98	1142	198
7A	1220	102	1147	202
8A	1250	102	1151	202
9A	1250	102	1151	200
10A	2200	108	1202	202

7. Analysis Technique

From each series of four negatives taken on each run, that one having the diffusing cluster nearest the center was selected for analysis. A line 4.5 inches long, positioned in a true North-South direction, was drawn through the center of the cluster on each of these negatives. The negatives were placed on a Travel-Graph transpaque overhead projector and projected onto a white cardboard sheet. The position of each particle was marked on the sheet as was the 4.5-inch line, resulting in an enlargement of the picture by a factor of about five. Each particle was assigned a number with which it was marked and thus identified throughout the series of enlargements.

A length scale was assigned to each negative based upon the relationship:

$$\frac{\text{image}}{\text{focal length}} = \frac{\text{ground covered}}{\text{altitude}}.$$

The camera used has an image size of 9 by 9-inches and a focal length of 12 inches. The altitude changed on each run. From the length scale the ground distance represented by the 4.5-inch line could be determined and thus the distance between particles could be measured on the enlargement. The distance between particles was measured to a sixteenth of an inch. The runs analyzed and the scales used are shown in Tables III and IV.

TABLE III

PAPER PATTERN

RUN	SCALE
1	1 inch = 14.822 feet
2	1 inch = 18.0325 feet
3	1 inch = 17.7515 feet
4	1 inch = 18.4615 feet
6	1 inch = 20.6915 feet
7	1 inch = 20.9847 feet

TABLE IV

CURRENT-CROSS PATTERN

RUN	SCALE
1A	1 inch = 19.4424 feet
2A	1 inch = 20.4489 feet
3A	1 inch = 18.6145 feet
4A	1 inch = 19.1278 feet
5A	1 inch = 19.7260 feet
6A	1 inch = 19.6990 feet

The forty-foot motor launch could have been used to determine a more accurate length scale. However, it appeared in only seven of the negatives. From these negatives it was determined that the plane's altimeter was registering from 8 to 18 feet too high.

Neighbor separations were measured without respect to their direction and without projection onto a coordinate axis. As was pointed out by Ozmidov (1956), "In reality, it is possible to imagine a case where one particle moves relative to another in a circular direction, that is $\Delta \ell = 0$, and there is no diffusion, while at the same time the projections of the segment $\Delta \ell$ on the axes of the coordinates are not equal to zero.... Hence we must regard as untrue the contention of Richardson and Stommel that it is possible to calculate the diffusion coefficient by measuring the projections of the distances between the diffusing particles."

The quantity ℓ_0 was measured for a specific neighbor pair on one enlargement and ℓ_1 was measured between the same pair on the enlargement corresponding to the next run. The scale of the diffusion, $\frac{1}{2}(\ell_0 + \ell_1)$, and the neighbor diffusivity, $\frac{(\ell_1 - \ell_0)^2}{2T}$, were calculated for each pair over the whole series of photographs. Time was recorded only to the nearest minute, resulting in a possible error of 60 seconds between pictures.

All the values of $\frac{1}{2}(\ell_0 + \ell_1)$ and $\frac{(\ell_1 - \ell_0)^2}{2T}$ were separated into similar groups of $\frac{1}{2}(\ell_0 + \ell_1)$. The class intervals for the different groups were chosen to be as small as possible while still allowing for thirty or more values within each class, making each class average as statistically meaningful as possible.

The two obvious sources of error in this analysis technique are the altitude error and the error inherent in recording the time to the nearest minute. The resulting systematic error is negligible. The random error might be important. The analysis of such error seems not to have been discussed in the literature and further work should probably incorporate such an analysis.

8. Results

The class intervals used, the number of values within each interval, and the results obtained from averaging these values are shown in Table V. The actual values within each group are contained in Appendix II.

TABLE V

PAPER SHEETS

CLASS INTERVAL (FEET)	NUMBER OF VALUES AVERAGED	RESULTS
50 - 100	36	$F(75.183) = .19855$
100 - 170	39	$F(141.92) = .36587$
150 - 200	31	$F(168.91) = .62068$
255 - 310	33	$F(283.55) = 1.1383$
300 - 400	36	$F(343.82) = 1.8259$
350 - 460	26	$F(397.58) = 2.0874$

TABLE V Cont.

CURRENT CROSSES

CLASS INTERVAL (FEET)	NUMBER OF VALUES AVERAGED	RESULTS
50 - 100	34	F(79.52) = .08159
80 - 140	38	F(105.4) = .10285
150 - 200	41	F(167.17) = .11983
300 - 350	46	F(325.65) = .29100
350 - 400	36	F(369.4) = .52743
400 - 490	34	F(457.16) = .76513

The resulting values of $\frac{1}{2}\overline{(\ell_0+\ell_1)}$ and $\frac{\overline{(\ell_1-\ell_0)^2}}{2T}$ for both the paper sheets and the current crosses were plotted on logarithmic paper as shown in Figure 2. Figure 2 also includes two lines of four-thirds slope that best fit the composite data considered by Olson and Ichiye (1959), Orlob (1959) and Ozmidov (1959). The dots represent the points determined by Stommel (1949) and the numbers in brackets above the dots indicate the number of measurements averaged to obtain each point. The results of Gunnerson's dye-stream experiments in Santa Monica Bay are plotted as letter D.

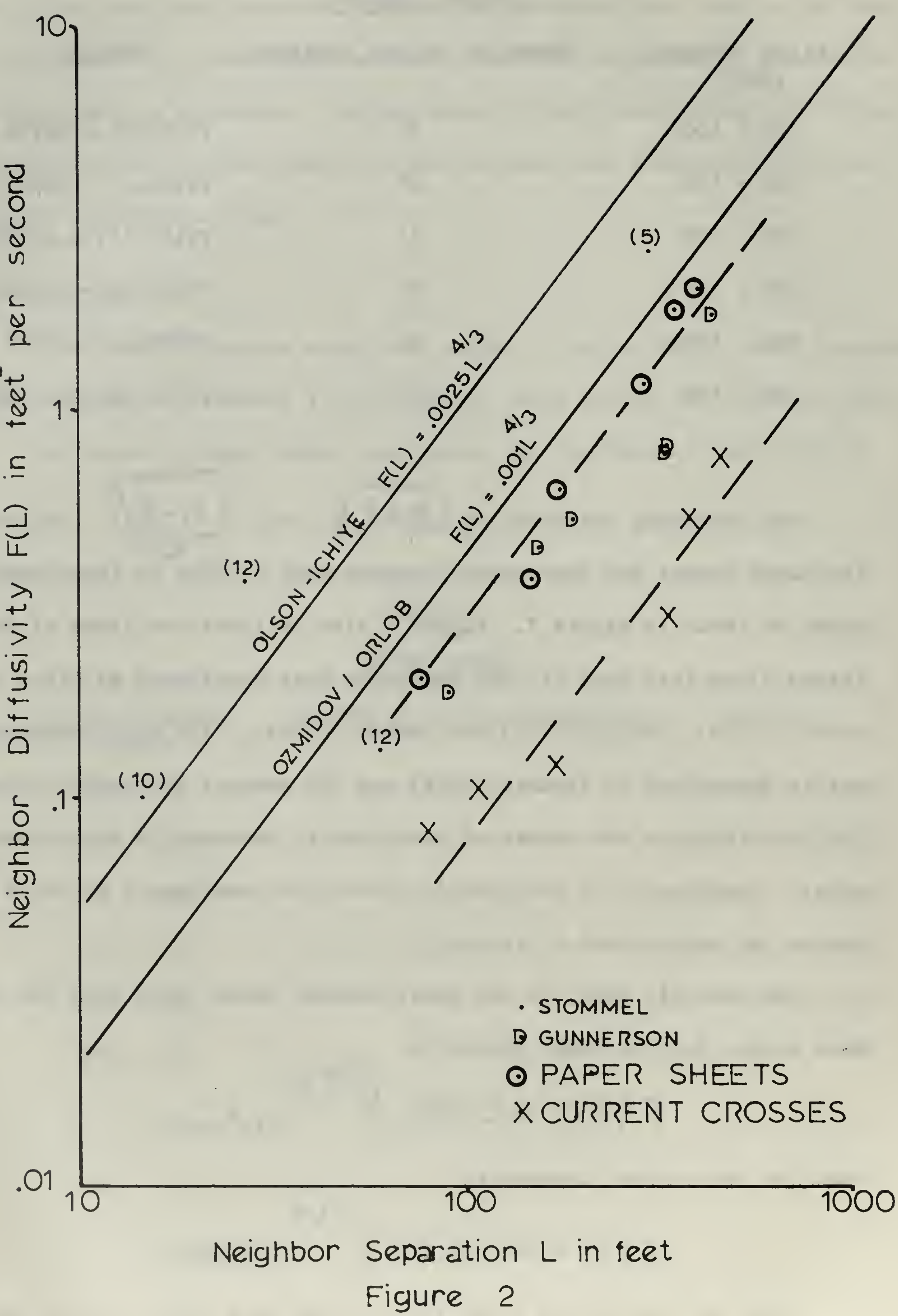
The best-fit line, in the least-squares sense, that fits the six data points for the paper sheets is

$$F(\ell) = 0.0032\,\ell^{1.46} \quad (\mathrm{ft}^2/\mathrm{sec});$$

that for the current crosses is

$$F(\ell) = 0.0029\,\ell^{1.24} \quad (\mathrm{ft}^2/\mathrm{sec}).$$

Lines of four-thirds slope also fit the data rather nicely, and are



10
1
.1
.01
10
100
1000
Neighbor Diffusivity F(L) in feet^2 per second
OLSON-ICHIYE F(L) = .0025L$^{4/3}$
OZMIDOV / ORLOB F(L) = .001L$^{4/3}$
(5)
(12)
(12)
(10)
· STOMMEL
D GUNNERSON
⊙ PAPER SHEETS
X CURRENT CROSSES
Neighbor Separation L in feet

Figure 2

plotted on Figure 2 as dashed lines. These lines represent the relationships:

$$F(\ell) = 0.00065\,\ell^{4/3} \quad (\mathrm{ft}^2/\mathrm{sec}) \quad \text{(paper sheets)}$$

$$F(\ell) = 0.00017\,\ell^{4/3} \quad (\mathrm{ft}^2/\mathrm{sec}) \quad \text{(current crosses).}$$

9. <u>Conclusions</u>

The data for both the paper sheets and the current crosses agree very well with the four-thirds law; a relationship of the sort

$$F(\ell) \propto \ell^{4/3}$$

fits the data with a good degree of accuracy in both cases. From this it can be concluded that the four-thirds law is applicable when relating the horizontal diffusion coefficients to the associated neighbor separations over the scale from 75 to 460 feet, at least in the area investigated.

It is also interesting to note the agreement between the data obtained from the paper sheets and the data obtained by Gunnerson (1959) using fluorescein dye streams released in water 180 feet deep in Santa Monica Bay. The investigations cover almost identical scale ranges and are in approximately the same depth of water (180 versus 216 feet) off the California coast. They yield very similar four-thirds relationships:

$$F(\ell) = 0.00065\,\ell^{4/3} \quad (\mathrm{ft}^2/\mathrm{sec})$$

as reported in this paper and

$$F(\ell) = 0.00051\,\ell^{4/3} \quad (\mathrm{ft}^2/\mathrm{sec})$$

as reported by Gunnerson.

The relationship

$$F(\ell) = 0.001\,\ell^{4/3} \quad (\mathrm{ft}^2/\mathrm{sec})$$

best describes the composite data considered by Orlob (1959) and the data obtained by Ozmidov (1959) at scales from about 1 to 66 feet and 1000 to 5000 feet. Figure 3 shows Ozmidov's (1959) large-scale data plotted along with the data from the paper sheets. All the data points shown represent averages of at least 26 separate measurements of $F(\ell)$ and ℓ.

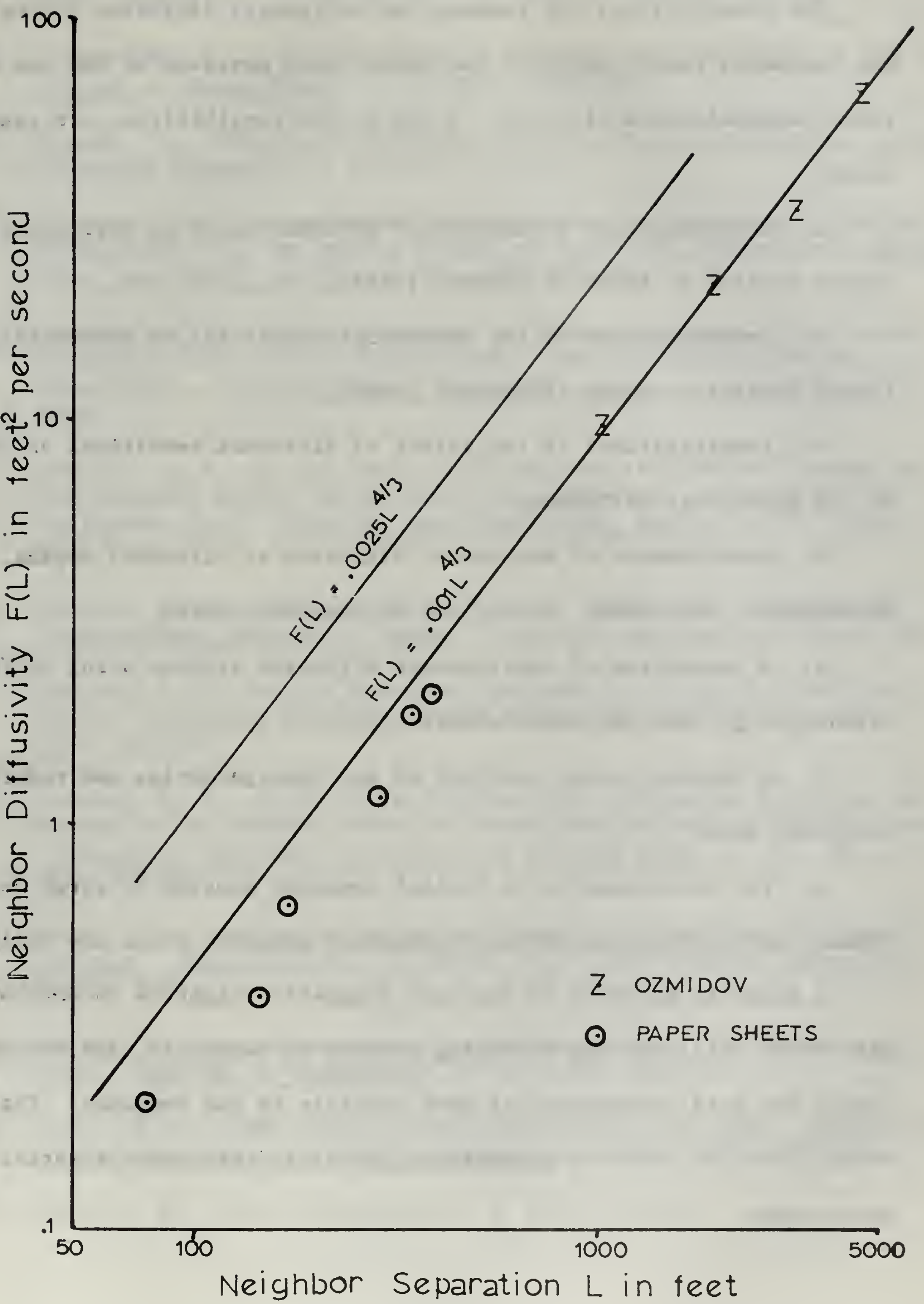
From consideration of the present results and those of Orlob (1959) and Ozmidov (1959) it would seem that k in the relationship

$$F(\ell) = k\ell^{4/3} \qquad (ft^2/sec)$$

is much closer to .001 ($ft^{2/3}/sec$) than to .00252 $\frac{ft^{2/3}}{sec}$ as found by Olson and Ichiye (1959). The difference is significant. The values for diffusivity arrived at by using the relationship of Olson and Ichiye for a given scale size are on the order of 2 to 3 times as large as those resulting from the relationship of Orlob and Ozmidov.

There is a significant difference between the value of k for the paper sheets and that for the current crosses. The reason for this difference could be that the horizontal diffusion is different below the surface or merely that the paper sheets diffuse differently than the current crosses due to the great difference in weight and configuration between the two. More experiments with the current crosses at different depths need to be conducted to gain even a partial answer to this question.

The method used to gather horizontal diffusion data seems applicable for obtaining data at scales much larger than those investigated. A three-dimensional view of the horizontal diffusion process is still to be achieved, but seems well within the capabilities of this method.



100
10
1
.1
50
100
1000
5000
Neighbor Diffusivity F(L) in feet² per second
Neighbor Separation L in feet
F(L) = .0025L 4/3
F(L) = .001L 4/3
Z OZMIDOV
⊙ PAPER SHEETS

Figure 3

10. Recommendations for further research

The possibilities for research on horizontal diffusion in the ocean are enormous, particularly at the scales made possible by the use of photo-reconnaissance aircraft. A few of the possibilities are suggested below:

a. Investigations of the effect of water depth on horizontal diffusion similar to those of Ozmidov (1957),

b. Investigations of the anisotropic qualities of horizontal diffusion similar to those of Ozmidov (1957),

c. Investigations of the effect of different conditions of stability on the horizontal diffusion,

d. Measurements of horizontal diffusion at different depths, as attempted in this paper, by the use of current crosses,

e. A comparison of simultaneous diffusion studies using different methods, e.g., dye and paper sheets,

f. A thorough error analysis of any data-gathering and reduction technique used,

g. The development of a digital computer program to carry out the tedious data reductions required when many neighbor pairs are analyzed.

A possible approach to the last suggestion might be to develop a grid to be laid over the diffusing pattern of particles, and thereby supply the grid coordinates of each particle to the computer. The grid origin could be fixed to a particular particle throughout a series of photographs.

BIBLIOGRAPHY

1. Richardson, L. F. Atmospheric Diffusion shown on a Distance-Neighbor Graph. Proceeding of the Royal Society of London, v. 110A, 1926: 709-737.

2. Richardson, L. F. and H. Stommel. Note on Eddy Diffusion in the Sea. Journal of Meteorology, v. 5, 1948: 238-240.

3. Stommel, H. Horizontal diffusion due to oceanic turbulence. Journal of Marine Research, v. 8, 1949: 199-225.

4. Ozmidov, R. V. Experimental investigation of horizontal turbulent diffusion in the sea and in a shallow artificial reservoir. Isvestia Akad. Nauk SSSR, geofiz., v. 6, 1957: 756-764.

5. Ozmidov, R. V. On the dependence of the coefficient of horizontal turbulent diffusion in wind-currents upon the relative depth of a water basin. Isvestia Akad. Nauk SSSR, geofiz, v. 8, 1959: 1242-1246.

6. Ozmidov, R. V. On the rate of dissipation of turbulent energy in sea-currents and on the dimensionless universal constant in the "4/3-power law." Isvestia Akad. Nauk SSSR, geofiz. v. 8, 1960: 1234-1237.

7. Ozmidov, R. V. Some aspects of oceanic turbulence. Journal of Oceanographic Society of Japan. Twentieth Anniversary Volume, 1962.

8. Ozmidov, R. V. On the turbulent exchange in a stably stratified ocean. Isvestia, Atmospheric and Oceanic Physics Series. v. 1, 1965: 853-860.

9. Ozmidov, R. V. Study of the Medium-Scale Horizontal Turbulent Exchange in the Ocean by Means of Radar Observations of Floating Buoys. Proceedings (Doklady) of Academy of Science USSR, v. 126, No. 1, 1959: 63-65.

10. Okubo, A. A Review of Theoretical Models of Turbulent Diffusion in the Sea. Chesapeake Bay Institute, The Johns Hopkins University, TR-30, 1962.

11. Taylor, G. The present position in the theory of turbulent diffusion. Advances in Geophysics, v. 6, 1959: 101-112.

12. Orlob, G. T. Eddy diffusion in homogeneous turbulence. Transactions of American Society of Civil Engineers, v. 126, 1959: 397-438.

13. Hill, M. N. (editor) The Sea, v. I, 1962: 817-823.

14. Gunnerson, C. G. Sewage disposal in Santa Monica Bay. Transactions of American Society of Civil Engineers. v. 124, 1959: 823-851.

15. Alsaffar, A. M. Lateral diffusion in a tidal estuary. Journal of Geophysical Research, v. 71, 1966: 5837-5841.

16. Kolmogoroff, A. N. The local structure of turbulence in incompressible fluids for very large Reynolds number, Compt. Rend. Acad. Sci. USSR, v. 30, 1941.

17. Isayeva, L. S. and I. L. Isayev. Horizontal Eddy Diffusion in the Sea. Soviet Oceanography, v. 28, 1963: 25-27.

18. Bourret, R. and S. Broida. Turbulent diffusion in the sea. Bulletin of Marine Science in the Gulf and Caribbean, v. 10, 1960: 355-367.

19. Ichiye, T. Oceanic Turbulence, The Oceanographic Institute of Florida State University, Unpublished Manuscript. Office of Naval Research Technical Paper No. 2, 1962.

20. Pochapsky, T. E. Diffusing particles in the deep ocean. (abstract) Symposium on Diffusion in Oceans and Fresh Waters. Lamont Geological Observatory, 1965: 114-115.

21. Foxworthy, J. E., R. B. Tibby, and G. Barsour. Multi-dimensional aspects of eddy diffusion determined by dye diffusion experiments in coastal waters (Summary). Symposium on Diffusion in Oceans and Fresh Waters. Lamont Geological Observatory, 1965: 61-73.

22. Foxworthy, J. E., G. M. Barsom and R. B. Tibby. On Eddy Diffusivity and the Four-Thirds Law in Highly Stratified Near-Shore Coastal Waters. Abstract. Transactions of the American Geophysical Union, v. 47, No. 3, 1966: 478.

23. Olson, F. C. W. and T. Ichiye. Horizontal Diffusion, Science, v. 130, 1959: 1255.

24. Obukov, A. M. Distribution of energy in the spectrum of a turbulent current. Isvestia Akad. Nauk. SSSR, v. 4-5, 1941.

25. Wiegel, R. L. Oceanographical Engineering, Prentice-Hall, 1964.

26. Neumann, G. and W. J. Pierson, Jr. Principles of Physical Oceanography, Prentice-Hall, 1966.

APPENDIX I

Description of current crosses, associated floats, and markers

The current crosses were designed to obtain diffusion data at any depth desired. The current cross, styrofoam float, and plywood marker are shown in Figure 4. The crosses were made of stainless steel and consisted of four perpendicular vanes. Each vane was 35 inches high and 6 inches wide. The cross weighed 9 pounds.

One half of the styrofoam packing of an expendable bathythermograph was used as a float for each current cross. A half-inch hole was drilled through the styrofoam float and an aluminum rod, with an aluminum plate 7 inches in diameter attached, was passed through this hole. The end of the rod had a small hole through which a line could be led and by which the float could be attached to the current cross. The plate served to hold the rod in place and also to anchor a marker that was placed on top of the float for identification from the air.

At first the 30.5 by 25.5-inch paper sheets used in the paper diffusion pattern were used as markers, but they tore too easily. After some experimenting, 2-foot by 4-foot marine plywood sheets, 1/4 of an inch thick and painted white, were used as markers. They each weighed 6 pounds.

The whole combination of current cross, float and plywood marker was almost neutrally buoyant when placed in the water; about 1/8 of an inch of the plywood marker was exposed to the wind, the rest of the combination was submerged. The vertical area of the float and plywood marker was 200 square inches while that of the current cross was 832 square inches. The current cross had more than four times the vertical area of the float and marker.

CURRENT CROSS, FLOAT AND MARKER

Figure 4

APPENDIX II

Values of ℓ_0, ℓ_1, and T used in computing

$$\ell = \tfrac{1}{2}\left(\ell_0 + \ell_1\right) \qquad F(\ell) = \frac{(\ell_1 - \ell_0)^2}{2T}$$

PAPER SHEETS
(50-100 feet)

ℓ_0 (feet)	ℓ_1 (feet)	T (minutes)	$F(\ell)$ (ft^2/sec)
73.56	78.89	3	.0789
41.65	54.10	3	.4306
52.29	52.97	3	.0013
57.61	54.10	3	.0342
63.81	78.90	3	.6317
84.19	94.67	3	.3051
83.31	91.29	3	.1769
74.45	94.67	3	1.1357
90.40	83.40	3	.1361
84.19	81.15	3	.0257
93.06	92.42	3	.0012
78.89	77.66	4	.0031
54.10	59.91	4	.0703
52.97	45.49	4	.1166
54.10	72.12	4	.6765
78.89	62.13	4	.5852
94.67	85.43	4	.1779
91.29	93.20	4	.0076
94.67	87.65	4	.1027
83.40	79.88	4	.0258
81.15	82.10	4	.0019
92.42	89.87	4	.0135
64.66	41.97	4	1.0726
45.26	48.53	4	.0227
80.18	90.50	4	.2219
67.25	82.63	4	.4928
80.18	76.07	4	.03519
59.91	65.77	5	.0572
72.12	85.39	5	.2937
62.13	56.54	5	.0521
85.43	75.00	5	.1813
93.20	92.31	5	.0013
87.65	79.62	5	.1075
79.88	76.15	5	.0232
82.10	91.15	5	.1366
89.87	91.15	5	.0027

(100-170 feet)

ℓ_0	ℓ_1	T	$F(\ell)$
107.34	118.34	3	.3423
93.06	105.94	3	.4608
109.01	107.07	3	.0104
119.64	134.12	3	.5824
133.82	112.96	3	1.2087
124.96	135.24	3	.2936
159.53	174.69	3	.6384
157.75	173.56	3	.6943
116.98	116.08	3	.0023
155.98	162.29	3	.1106
155.09	167.93	3	.4580
156.87	162.29	3	.0816
118.34	114.28	4	.0345
105.94	102.07	4	.0312
107.07	117.60	4	.2310
134.12	134.25	4	.0000
112.96	149.78	4	2.8244
135.24	146.45	4	.2618
174.69	161.98	4	.3365
173.56	167.53	4	.0755
116.08	122.04	4	.0740
162.29	157.54	4	.0470
167.93	163.09	4	.0488
162.29	159.76	4	.0133
177.17	144.27	4	2.2550
151.31	159.6	4	.5720
162.95	178.37	4	.4954
114.28	140.77	5	1.1695
102.07	101.54	5	.0005
163.09	160.38	5	.0122
159.76	155.77	5	.0265
117.60	130.38	5	.2727
134.25	140.77	5	.0708
149.78	152.31	5	.0107
177.52	166.15	5	.2155
146.45	154.62	5	.1112
161.98	148.85	5	.2873
167.53	169.62	5	.0072
122.04	124.62	5	.0112
177.17	144.27	5	

(150-200 feet)

ℓ_0	ℓ_1	T	$F(\ell)$
163.07	183.71	3	1.1833
166.62	193.85	3	2.0596
159.53	174.69	3	.6384
157.75	173.56	3	.6943
155.98	162.29	3	.1106
155.09	167.93	3	.4580
156.87	162.29	3	.0816
183.71	177.52	3	.0798
193.85	193.05	4	.0013
174.69	161.98	4	.3365
173.56	167.53	4	.0755
162.29	157.54	4	.0470
167.93	163.09	4	.0488
162.29	159.76	4	.0133
177.17	144.27	4	2.2550
152.6	179.91	4	6.2153
151.31	167.88	4	.5720
164.24	182.30	4	.6795
173.29	175.75	4	.0126
174.59	186.24	4	.2828
162.95	178.37	4	.4954
177.17	187.55	4	.2245
149.78	152.31	5	.0107
177.52	166.15	5	.2155
146.45	154.62	5	.1112
193.05	204.23	5	.2080
161.98	148.85	5	.2873
167.53	169.62	5	.0072
157.54	190.38	5	1.7975
163.09	160.39	5	.0122
159.76	155.77	5	.0265

(255-310 feet)

ℓ_0	ℓ_1	T	$F(\ell)$
276.51	302.04	3	1.8105
255.35	277.25	3	1.3323
266.76	314.44	3	6.3150
272.08	299.79	3	2.3290
264.10	287.34	3	1.5000
247.26	268.23	3	1.2215
233.08	286.27	3	7.8588
302.04	298.45	4	.0269
277.25	271.82	4	.0614
299.79	299.56	4	.0001
287.39	284.02	4	.0230
268.23	266.27	4	.0080
286.27	296.23	4	.2067
292.27	323.95	4	2.0909
298.45	293.08	5	.0480
271.82	280.38	5	.1221
299.56	309.23	5	.1558
284.02	290.77	5	.0760
266.27	285.00	5	.5847
296.23	301.15	5	.0403
276.51	298.45	7	.5731
254.35	271.82	7	.3633
266.76	321.75	7	3.5999
272.08	299.56	7	.8999
264.10	284.02	7	.4724
247.26	266.27	7	.4508
233.08	296.23	7	4.7475
302.04	293.08	9	.0743
277.25	280.38	9	.0091
299.79	309.23	9	.0825
287.39	290.77	9	.0106
268.23	285.00	9	.2604
286.27	301.15	9	.2050

(300-400 feet)

ℓ_0	ℓ_1	T	$F(\ell)$
366.91	409.11	3	4.9467
343.87	380.94	3	3.8172
319.05	353.89	3	3.3717
299.55	355.01	3	8.5439
296.01	331.35	3	3.4692
380.94	377.22	4	.0288
353.89	349.48	4	.0405
355.01	363.91	4	.1650
331.35	329.51	4	.0071
302.04	298.45	4	.0269
314.44	321.75	4	.1113
397.02	323.95	4	11.1234
318.13	331.82	4	.3905
328.48	338.38	4	.2042
376.33	392.15	4	.5214
351.76	308.21	4	3.9513
292.27	323.95	4	2.0909
307.79	334.44	4	1.4796
358.22	320.02	4	3.0400
377.22	406.15	5	1.3948
349.48	393.46	5	3.2237
363.91	369.23	5	.0472
329.51	331.15	5	.0045
321.75	335.77	5	.3277
299.56	309.23	5	.1558
366.91	406.07	7	1.8247
343.86	377.22	7	1.3241
319.05	349.48	7	1.1023
299.55	363.91	7	4.9312
296.01	329.51	7	1.3360
380.94	406.15	9	.5885
353.89	393.46	9	1.4498
355.01	369.23	9	.1872
331.35	331.15	9	.0001
314.44	335.77	9	.4213
299.79	309.23	9	.0825

(350-460 feet)

l_0	l_1	T	$F(l)$
371.34	431.65	3	10.1036
366.91	409.11	3	4.9467
343.87	380.94	3	3.8172
431.65	436.02	4	.0398
409.11	406.07	4	.0198
380.94	377.22	4	.0288
353.89	349.48	4	.0405
355.01	363.91	4	.1650
451.33	461.66	4	.2223
460.39	468.22	4	.1277
430.64	410.51	4	.8442
397.02	323.95	4	11.1234
376.33	392.15	4	.5214
436.02	474.23	5	2.4333
406.07	441.92	5	2.1432
377.22	406.15	5	1.3948
349.48	393.46	5	3.2237
363.91	369.23	5	.0472
366.34	436.02	7	4.9804
366.91	406.07	7	1.8247
343.87	377.22	7	1.3241
431.65	474.23	9	1.6788
409.11	441.92	9	.9968
380.94	406.15	9	.5885

CURRENT CROSSES

(50-100 feet)

ℓ_0 (feet)	ℓ_1 (feet)	T (minutes)	$F(\ell)$ (ft^2/sec)
55.90	58.79	3	.0232
99.64	99.69	3	.0000
75.34	75.41	3	.0000
76.56	79.24	3	.0201
88.71	90.74	3	.0114
94.78	95.85	3	.0032
54.68	49.84	3	.0651
82.63	84.35	3	.0082
97.21	102.25	3	.0703
70.48	66.46	3	.0449
59.77	61.64	3	.0097
78.90	81.37	3	.0169
89.66	88.77	3	.0022
56.19	64.11	3	.1742
58.58	72.74	3	.5570
66.95	69.04	3	.0121
60.32	59.77	5	.0005
83.51	78.90	5	.0354
96.27	89.66	5	.0728
49.88	56.19	5	.0664
52.20	58.58	5	.0679
72.89	66.95	5	.0588
61.64	75.10	5	.3020
81.37	87.41	5	.0608
96.16	89.88	5	.0657
72.74	91.11	5	.5624
69.04	72.64	5	.0216
58.79	60.32	7	.0027
79.24	83.51	7	.0217
90.74	102.07	7	.1529
95.85	96.27	7	.0002
84.35	95.11	7	.1646
102.25	96.27	7	.0424
66.46	72.89	7	.0494

(80-140 feet)

ℓ_0	ℓ_1	T	$F(\ell)$
98.43	109.91	3	.3661
99.64	99.69	3	.0000
114.22	109.91	3	.0516
88.71	90.74	3	.0114
94.78	95.85	3	.0032
133.67	138.03	3	.0528
82.63	84.35	3	.0082
97.21	102.25	3	.0703
109.03	117.16	5	.1101
83.51	78.90	5	.0354
102.07	113.57	5	.2204
102.07	105.20	5	.0163
96.27	89.66	5	.0728
133.39	138.68	5	.0466
95.11	111.18	5	.4303
96.27	104.01	5	.0997
123.29	134.20	5	.1984
118.36	124.35	5	.0598
110.96	121.89	5	.1991
120.82	123.12	5	.0088
96.16	89.88	5	.0657
72.74	91.11	5	.5624
109.91	109.03	7	.0008
79.24	83.51	7	.0217
109.91	102.07	7	.0732
90.74	102.07	7	.1529
95.85	96.27	7	.0002
138.03	133.39	7	.0256
84.35	95.11	7	.1646
102.35	96.27	7	.0424
117.16	123.29	3	.1044
78.90	81.37	3	.0169
113.50	118.36	3	.0637
105.20	110.96	3	.0694
89.66	88.77	3	.0022
138.68	140.55	3	.0097
111.18	120.82	3	.2581
104.01	96.16	3	.1712

(150-200 feet)

ℓ_0	ℓ_1	T	$F(\ell)$
155.54	167.43	3	.3927
162.83	163.59	3	.0016
149.46	152.09	3	.0192
188.35	186.60	3	.0085
177.41	186.60	3	.2346
160.40	162.31	3	.0101
153.11	153.37	3	.0002
179.84	176.37	3	.0334
172.55	158.48	3	.5499
148.47	169.75	5	.7555
151.95	156.61	5	.0362
183.27	190.08	5	.0775
155.43	151.83	5	.0216
182.11	194.87	5	.2713
151.95	159.00	5	.0821
148.47	162.59	5	.3322
154.27	154.22	5	.0000
187.91	187.69	5	.0000
151.95	153.02	5	.0019
169.76	168.90	5	.0021
156.61	161.51	5	.0667
190.08	197.26	5	.1436
151.83	155.34	5	.0342
194.86	199.73	5	.0659
159.00	161.51	5	.0175
162.59	168.90	5	.1106
154.22	149.18	5	.0706
187.69	188.63	5	.0025
168.90	177.29	5	.1173
161.51	167.44	5	.0586
161.51	171.14	5	.1546
168.90	180.98	5	.2432
167.43	148.47	7	.4252
163.59	151.95	7	.1680
186.60	183.27	7	.0130
149.53	155.43	7	.0414
186.60	182.11	7	.0242
162.31	151.95	7	.1288
153.37	154.27	7	.0010
176.37	187.91	7	.1574
158.48	151.95	7	.0519

(300-350 feet)

ℓ_0	ℓ_1	T	$F(\ell)$
306.22	318.24	3	.4013
309.86	316.96	3	.1400
337.81	350.19	3	.4257
319.58	331.02	3	.3635
297.71	306.73	3	.2260
332.95	345.08	3	.4087
328.09	337.41	3	.2413
319.20	323.01	3	.0403
316.80	324.25	3	.1542
341.91	355.07	3	.4811
316.80	325.48	3	.2091
308.54	313.87	5	.1894
301.58	316.80	5	.3861
336.37	352.67	5	.4428
321.29	341.91	5	.7086
306.22	316.80	5	.1866
323.01	338.58	5	.4040
324.25	337.35	5	.2860
318.24	308.54	7	.1120
316.96	301.58	7	.2816
350.19	336.37	7	.2274
331.02	321.29	7	.1127
306.73	306.22	7	.0003
345.08	346.81	7	.0036
337.41	351.45	7	.2347
308.54	323.01	8	.2183
301.58	324.25	8	.5353
336.37	362.47	8	.7092
321.29	355.07	8	1.1882
306.22	325.48	8	.3865
319.20	338.58	8	.3912
316.80	337.35	8	.4399
316.80	343.50	8	.7426
306.22	309.47	10	.0005
319.59	354.84	10	1.0356
309.86	302.49	10	.0450
337.81	337.39	10	.0002
319.59	332.26	10	.0060
297.71	307.14	10	.0741
332.95	347.86	10	.1852
328.09	343.20	10	.1903
318.24	319.20	12	.0007
316.96	316.81	12	.0000
331.02	341.91	12	.0825
306.73	316.80	12	.0708
337.41	362.23	12	.4270

(350-400 feet)

ℓ_0	ℓ_1	T	$F(\ell)$
346.32	361.69	3	.6562
354.82	320.64	3	.6952
376.58	385.89	3	.2408
376.58	385.89	3	.2408
352.67	362.47	3	.2668
359.84	371.10	3	.3522
362.23	377.26	3	.6275
364.21	376.58	5	.2550
358.41	376.58	5	.5502
346.81	359.84	5	.2830
351.45	362.23	5	.1937
385.89	412.45	5	1.1757
385.89	407.52	5	.7798
362.47	380.44	5	.5382
355.07	370.59	5	.4015
325.48	343.50	5	.5412
371.10	397.67	5	1.1766
377.26	390.29	5	.2830
361.69	364.21	7	.0076
370.64	358.41	7	.1781
364.21	385.89	8	.4896
358.41	385.89	8	.7865
346.81	371.10	8	.6142
351.45	377.26	8	.6938
376.58	412.45	8	1.3403
376.58	407.52	8	.9971
352.67	380.44	8	.8033
341.91	370.59	8	.8568
359.84	397.67	8	1.4907
362.23	390.29	8	.8202
346.32	365.31	10	.3005
354.82	359.49	10	.0182
361.69	376.58	12	.1542
370.64	376.58	12	.0250
350.19	352.67	12	.0043
345.08	359.84	12	.1500

(400-490 feet)

ℓ_0	ℓ_1	T	$F(\ell)$
397.35	406.42	3	.2285
442.31	460.10	3	.8791
422.87	437.10	3	.5625
396.14	410.26	3	.5538
402.49	420.81	5	.5594
459.32	480.59	5	.7540
437.29	457.87	5	.7059
417.57	442.33	5	1.0218
420.81	435.21	5	.5760
480.59	495.62	5	.6275
457.87	473.42	5	.6717
442.23	462.33	5	1.1223
435.21	460.47	5	1.0634
473.42	489.11	5	1.6401
462.33	483.86	5	.7726
406.42	402.49	7	.0184
460.10	459.32	7	.0007
437.10	437.29	7	.0000
410.26	417.57	7	.0636
402.49	435.21	8	1.1150
459.32	495.62	8	1.3720
437.29	473.43	8	1.3604
417.57	462.33	8	2.0870
420.81	460.47	8	1.6385
457.87	504.79	8	2.2931
442.33	483.86	8	1.7966
397.53	404.86	10	.0470
442.32	464.20	10	.3991
422.87	438.60	10	.2061
396.14	418.83	10	.4288
406.42	420.81	12	.1440
460.10	480.59	12	.2919
437.10	457.87	12	.3004
410.26	442.33	12	.7111

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13. ABSTRACT

This paper reports the results of field tests of Richardson's "four-thirds law" of horizontal diffusion, which relates the horizontal diffusion coefficients to particle separation, or eddy scale.

A U. S. Navy photo-reconnaissance aircraft was used to gather data of diffusing patterns of rectangular paper sheets on the ocean surface and of current crosses nine feet below the surface. The data were gathered in Monterey Bay, California in water thirty-six fathoms deep in an area approximately one and one-half miles from the shore. The scale covered was from seventy to four hundred and sixty feet. A two-particle analysis suggested by Richardson and Stommel was applied. The four-thirds law was found to be applicable to the horizontal diffusion analyzed in this investigation.

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